

UNITU-THEP-7/1999

hep-ph/9907563

Octet and Decuplet Baryons in a Confining and Covariant Diquark-Quark Model*

R. Alkofer, S. Ahlig, C. Fischer, M. Oettel, and H. Reinhardt

Institut für Theoretische Physik der Universität Tübingen
Auf der Morgenstelle, 72076 Tübingen, Germany

We treat baryons as bound states of scalar or axialvector diquarks and a constituent quark which interact through quark exchange. We obtain fully four-dimensional wave functions for both octet and decuplet baryons as solutions of the corresponding Bethe-Salpeter equation. Applications currently under investigation are: electromagnetic and strong form factors and strangeness production processes.

1. Motivation

Different types of hadronic models describe various aspects of baryon physics. Among them are nonrelativistic quark models, various sorts of bag models and approaches describing baryons by means of collective variables like topological or non-topological solitons [1]. Most of these models are designed to work in the low energy region and generally do not match the calculations within perturbative QCD. Considering the great experimental progress in the medium energy range [2], there is a high demand for models describing baryon physics in this region that connects the low and high energy regimes. To make progress in this direction we investigate a covariant formulation of a diquark-quark-model of baryons.

Our motivation to choose such an approach is fed from two sources. On the one hand, when starting with the fully relativistic Faddeev equation for bound states of three quarks, diquarks appear as effective degrees of freedom. These diquarks stand for correlated quark-quark pairs inside baryons. Thus they should not be confused with the notion of diquark condensates in the context of colour superconductivity. On the other hand, diquarks as constituents of baryons are naturally obtained when one starts with an NJL-type of model of colour octet flavour singlet quark currents [3]. Although in the limit $N_c \rightarrow \infty$ baryons emerge as solitons of meson fields [4], it can be shown for the case of three colours that both effects, binding through quark exchange in the diquark-quark picture and through mesonic effects, contribute equally [5].

*Supported by the BMBF (06-TU-888) and by the DFG (We 1254/4-1).
Talk given by R. Alkofer at PANIC 99.

Table 1

Components of the octet baryon wave function with their respective spin and orbital angular momentum. $(\gamma_5 C)$ corresponds to scalar and $(\gamma^\mu C)$, $\mu = 1 \dots 4$, to axialvector diquark correlations. Note that the partial waves in the first row possess a non-relativistic limit. See [6] for further details.

“non-relativistic” partial waves	$\begin{pmatrix} \chi \\ 0 \end{pmatrix}_{(\gamma_5 C)}$	$\hat{P}^4 \begin{pmatrix} 0 \\ \chi \end{pmatrix}_{(\gamma^4 C)}$	$\begin{pmatrix} i\sigma^i \chi \\ 0 \end{pmatrix}_{(\gamma^i C)}$	$\begin{pmatrix} i\left(\hat{p}^i(\vec{\sigma}\vec{p}) - \frac{\sigma^i}{3}\right)\chi \\ 0 \end{pmatrix}_{(\gamma^i C)}$
spin	1/2	1/2	1/2	3/2
orbital angular momentum	s	s	s	d
“relativistic” partial waves	$\begin{pmatrix} 0 \\ \vec{\sigma}\vec{p}\chi \end{pmatrix}_{(\gamma_5 C)}$	$\hat{P}^4 \begin{pmatrix} (\vec{\sigma}\vec{p})\chi \\ 0 \end{pmatrix}_{(\gamma^4 C)}$	$\begin{pmatrix} 0 \\ i\sigma^i(\vec{\sigma}\vec{p})\chi \end{pmatrix}_{(\gamma^i C)}$	$\begin{pmatrix} 0 \\ i\left(p^i - \frac{\sigma^i(\vec{\sigma}\vec{p})}{3}\right)\chi \end{pmatrix}_{(\gamma^i C)}$
spin	1/2	1/2	1/2	3/2
orbital angular momentum	p	p	p	p

2. Solving the four-dimensional Bethe-Salpeter equation: masses and wave functions

Starting from the Faddeev equation, one can approximate the two-quark irreducible T -matrix by separable contributions that can be viewed as loosely bound diquarks. The full three-body problem reduces then to a two-body one, in which bound states appear as the solution of a homogeneous Bethe-Salpeter equation. The attractive interaction between quark and diquark is hereby provided by quark exchange. In [6] we formalize this procedure by an effective Lagrangian containing constituent quark, scalar diquark and axialvector diquark fields. This leads to a coupled set of Bethe-Salpeter equations for octet and decuplet baryons.

We avoid unphysical thresholds by an effective parameterization of confinement in the quark and diquark propagators. We then solve the complete four-dimensional Bethe-Salpeter equation in ladder approximation and obtain wave functions for the octet and decuplet baryons [6]. The Lorentz invariance of our model has been checked explicitly by choosing different frames.

The implementation of the appropriate Dirac and Lorentz representations of the quark and diquark parts of the wave functions leads to a unique decomposition in the rest frame of the baryon. Besides the well known s -wave and d -wave components of non-relativistic formulations of the baryon octet we additionally obtain non-negligible p -wave contributions which demonstrates again the need for covariantly constructed models. Table 1 summarizes the structure of the octet wave function. Each of the eight components is to be multiplied with a scalar function which is given in terms of an expansion in hyper-spherical harmonics and is computed numerically.

In order to obtain the mass spectra for the octet and decuplet baryons we explicitly break SU(3) flavour symmetry by a higher strange quark constituent mass. Using the nucleon and the delta mass as input our calculated mass spectra [6] are in good agreement

Table 2

Octet and decuplet masses obtained with two different parameter sets. Set I represents a calculation with weakly confining propagators, Set II with strongly confining propagators, see [6]. All masses are given in GeV.

	m_u	m_s	M_Λ	M_Σ	M_Ξ	M_{Σ^*}	M_{Ξ^*}	M_Ω
Set I	0.5	0.65	1.123	1.134	1.307	1.373	1.545	1.692
Set II	0.5	0.63	1.133	1.140	1.319	1.380	1.516	1.665
Exp.			1.116	1.193	1.315	1.384	1.530	1.672

with the experimental ones, see Table 2. The wave functions for baryons with distinct strangeness content but same spin differ mostly due to flavour Clebsch-Gordan coefficients, the respective invariant functions being very similar. Due to its special role among the other baryons, we investigated the Λ hyperon in more detail and discussed its vertex amplitudes. In our approach, the Λ acquires a small flavour singlet admixture which is absent in $SU(6)$ symmetric non-relativistic quark models.

3. Applications: Form Factors and Strangeness Production

A significant test and a first application of our model is the calculation of various form factors [7,8]. The most important ingredient are the fully four-dimensional wave functions described above. It turns out that already the electromagnetic form factors of the nucleon provide severe restrictions for the parameters of the model.

The pion-nucleon form factor is calculated in view of its possible use in a spectator model for nucleon-nucleon scattering processes. At the soft point, $Q^2 = 0$, we find good agreement with experiment. In the spacelike region our $g_{\pi NN}$ falls like a monopole with a large cutoff used also in One-Boson-Exchange (OBE) models. Compared with a calculation including only scalar diquarks [7] we find a lower value for the pion-nucleon coupling at the soft point. Serving as a central ingredient for strangeness production processes the kaon-nucleon-lambda form factor $g_{KN\Lambda}$ is an issue of special interest. Due to flavour algebra the isospin configuration of the Λ singles out the scalar diquark as the only diquark contributing to nucleon-lambda transitions. With a pseudoscalar kaon not coupling to the scalar diquark we find ourselves in a comfortable position to handle such transitions. Results can be found in a forthcoming paper [8].

Going beyond the calculation of form factors we work on the application of our approach to production processes like kaon photoproduction and the associated strangeness production close to threshold. Great experimental progress in the last few years gave access to kinematical as well as some spin observables due to self-analyzing Λ decay. We try to reproduce these observables in our model with the aim to get further insight into the mechanisms of strangeness production inside the nucleon.

In the case of kaon photoproduction, $\gamma p \rightarrow K\Lambda$, preliminary results are gained by computing the left diagram of fig. 1 and the corresponding one with photon and kaon line crossed. Although the outcome of the total cross section is encouraging, the result

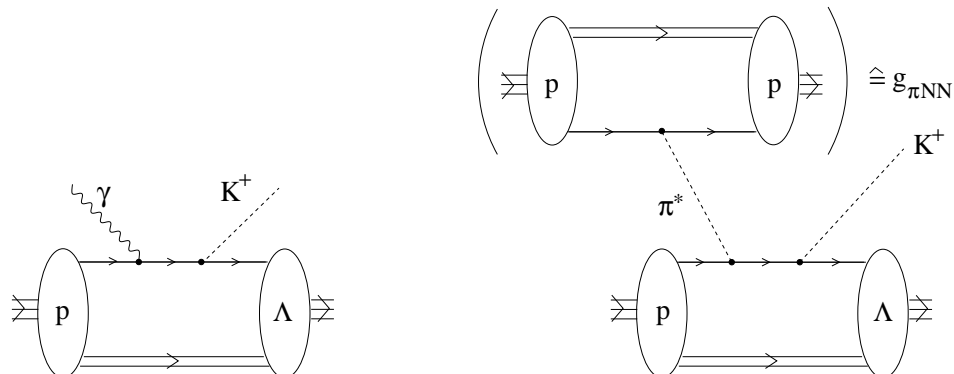


Figure 1. Typical diagrams to be computed in the diquark spectator picture. Left: kaon photoproduction. Right: strangeness production in proton-proton collisions.

for the polarisation asymmetries of the Λ falls too short.

Additionally, we describe the process $pp \rightarrow pK\Lambda$ employing an OBE-picture for internuclear forces. From our viewpoint, the diquarks act as mere spectators, so that the exchanged bosons couple only to the valence quarks of the baryons. In a first step we investigate the contributions stemming from pseudoscalar kaon and pion interchange between these two protons. The latter process is depicted in the right diagram of fig. 1. If instead of the pion a kaon is exchanged, the upper part of this diagram changes to the form factor $g_{KN\Lambda}$.

Possible applications are not exhausted by these examples. The diquark spectator picture can be extended to, *e.g.*, real and virtual Compton scattering. The quark structure functions (see also [9]), which are obtained from the imaginary part of the virtual Compton scattering amplitude, would allow a comparison with the results of perturbative calculations.

Acknowledgement:

R. A. thanks W. Bentz and P. Maris for interesting discussions and the organizers and conveners of PANIC 99.

REFERENCES

1. *see e.g.* R. Bhaduri, Models of the Nucleon, Addison-Wesley, New York, 1988.
2. *see e.g. these proceedings*
3. R. Alkofer, H. Reinhardt, Chiral Quark Dynamics, Springer Verlag (1995);
see also: H. Reinhardt, Phys. Lett. **B244** (1990) 316.
4. R. Alkofer *et al.*, Phys. Rep. **265** (1996) 139.
5. U. Zückert *et al.*, Phys. Rev. C **55** (1997) 2030.
6. M. Oettel *et al.*, Phys. Rev. C **58** (1998) 2459.
7. G. Hellstern *et al.*, Nucl. Phys. A 627 (1997) 679.
8. S. Ahlig *et al.*, in preparation.
9. K. Kusaka *et al.*, Phys. Rev. D **55** (1997) 5299.